LUYỆN TẬP (2 tiết)

Gợi ý trả lời câu hỏi và bài tập

46. a) $\sin(2\alpha + \alpha) = \sin 2\alpha \cos\alpha + \cos 2\alpha \sin\alpha = 2\sin\alpha\cos^2\alpha + (1 - 2\sin^2\alpha)\sin\alpha$

$= 2\sin\alpha(\cos^2\alpha - \sin^2\alpha) + \sin\alpha = 2\sin\alpha(1 - 2\sin^2\alpha) + \sin\alpha$

$= 3\sin\alpha - 4\sin^3\alpha.$

$\cos(2\alpha + \alpha) = \cos 2\alpha\cos\alpha - \sin 2\alpha\sin\alpha = (2\cos^2\alpha - 1)\cos\alpha - 2\sin^2\alpha\cos\alpha$

$= -\cos\alpha + 2\cos\alpha(\cos^2\alpha - \sin^2\alpha) = -\cos\alpha + 2\cos\alpha(2\cos^2\alpha - 1)$

$= 4\cos^3\alpha - 3\cos\alpha.$

b) $\sin\alpha\sin\left(\frac{\pi}{3} - \alpha\right)\sin\left(\frac{\pi}{3} + \alpha\right) = \sin\alpha\left(\sin^2\frac{\pi}{3}\cos^2\alpha - \sin^2\alpha\cos^2\frac{\pi}{3}\right) =$

$= \sin\alpha\left(\frac{3}{4}\cos^2\alpha - \frac{1}{4}\sin^2\alpha\right) = \frac{\sin\alpha}{4}(3 - 4\sin^2\alpha)$

$= \frac{1}{4}(3\sin\alpha - 4\sin^3\alpha) = \frac{1}{4}\sin 3\alpha$;

$$\cos\alpha\cos\left(\frac{\pi}{3}-\alpha\right)\cos\left(\frac{\pi}{3}+\alpha\right)=\cos\alpha\left(\cos^2\frac{\pi}{3}\cos^2\alpha-\sin^2\frac{\pi}{3}\sin^2\alpha\right)$$

$$=\cos\alpha\left(\frac{1}{4}\cos^2\alpha-\frac{3}{4}\sin^2\alpha\right)=\frac{\cos\alpha}{4}[\cos^2\alpha-3(1-\cos^2\alpha)]$$

$$=\frac{\cos\alpha}{4}(4\cos^2\alpha-3)=\frac{1}{4}\cos 3\alpha.$$

Ứng dụng

- $\sin 20^o\sin 40^o\sin 80^o=\sin 20^o\sin(60^o-20^o)\sin(60^o+20^o)$

$$=\frac{1}{4}\sin(3.20^o)=\frac{1}{4}\sin 60^o=\frac{\sqrt{3}}{8}.$$

- Tương tự, $\cos 20^o\cos 40^o\cos 80^o=\frac{1}{4}\cos(3.20^o)=\frac{1}{8}.$

Vậy $\tan 20^o\tan 40^o\tan 80^o=\sqrt{3}.$

47. a) $\cos 10^o\cos 50^o\cos 70^o=\cos 10^o\left[\frac{1}{2}(\cos 120^o+\cos 20^o)\right]=$

$$=-\frac{1}{4}\cos 10^o+\frac{1}{2}\cos 10^o\cos 20^o=-\frac{1}{4}\cos 10^o+\frac{1}{4}(\cos 30^o+\cos 10^o)$$

$$=\frac{1}{4}\cos 30^o=\frac{\sqrt{3}}{8}.$$

$\sin 20^o\sin 40^o\sin 80^o=\cos 70^o\cos 50^o\cos 10^o=\frac{\sqrt{3}}{8}.$

b) $\sin 10^o\sin 50^o\sin 70^o=\frac{1}{2}(\cos 20^o-\cos 120^o)\sin 10^o=$

$$=\frac{1}{4}\sin 10^o+\frac{1}{2}\sin 10^o\cos 20^o=\frac{1}{4}\sin 10^o+\frac{1}{4}(\sin 30^o-\sin 10^o)$$

$$=\frac{1}{4}\sin 30^o=\frac{1}{8}.$$

$\cos 20^o\cos 40^o\cos 80^o=\sin 70^o\sin 50^o\sin 10^o=\frac{1}{8}.$

48. Đặt $A = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$ thì khi cộng vế với vế ba đẳng thức

$$\cos\frac{2\pi}{7}\sin\frac{\pi}{7} = \frac{1}{2}\left(\sin\frac{3\pi}{7} - \sin\frac{\pi}{7}\right),$$

$$\cos\frac{4\pi}{7}\sin\frac{\pi}{7} = \frac{1}{2}\left(\sin\frac{5\pi}{7} - \sin\frac{3\pi}{7}\right),$$

$$\cos\frac{6\pi}{7}\sin\frac{\pi}{7} = \frac{1}{2}\left(\sin\pi - \sin\frac{5\pi}{7}\right),$$

ta được $A\sin\frac{\pi}{7} = -\frac{1}{2}\sin\frac{\pi}{7}$. Từ đó $A = -\frac{1}{2}$.

Cách khác. Cộng vế với vế ba đẳng thức

$$\cos\frac{2\pi}{7}\sin\frac{2\pi}{7} = \frac{1}{2}\sin\frac{4\pi}{7},$$

$$\cos\frac{4\pi}{7}\sin\frac{2\pi}{7} = \frac{1}{2}\left(\sin\frac{6\pi}{7} - \sin\frac{2\pi}{7}\right),$$

$$\cos\frac{6\pi}{7}\sin\frac{2\pi}{7} = \frac{1}{2}\left(\sin\frac{8\pi}{7} - \sin\frac{4\pi}{7}\right),$$

ta được $A\sin\frac{2\pi}{7} = -\frac{1}{2}\sin\frac{2\pi}{7} + \frac{1}{2}\left(\sin\frac{8\pi}{7} + \sin\frac{6\pi}{7}\right) = -\frac{1}{2}\sin\frac{2\pi}{7}$

(vì $\frac{8\pi}{7} + \frac{6\pi}{7} = 2\pi$ nên $\sin\frac{8\pi}{7} + \sin\frac{6\pi}{7} = 0$). Từ đó $A = -\frac{1}{2}$.

49. a) Ta có

$$\begin{aligned}
&\cos^2(\alpha + x) + \cos^2 x - 2\cos\alpha\cos x\cos(\alpha + x) = \\
&= \cos(\alpha + x)[\cos(\alpha + x) - 2\cos\alpha\cos x] + \cos^2 x \\
&= \cos(\alpha + x)(-\cos\alpha\cos x - \sin\alpha\sin x) + \cos^2 x \\
&= -\cos(\alpha + x)\cos(\alpha - x) + \cos^2 x = -\frac{1}{2}(\cos 2\alpha + \cos 2x) + \cos^2 x \\
&= -\frac{1}{2}\cos 2\alpha - \frac{\cos 2x}{2} + \cos^2 x = -\frac{1}{2}\cos 2\alpha + \frac{1}{2} \\
&= \sin^2\alpha\text{, không phụ thuộc vào } x.
\end{aligned}$$

b) Ta có $\sin 4x \sin 10x = \frac{1}{2}(\cos 6x - \cos 14x)$,

$$-\sin 11x \sin 3x = \frac{1}{2}(\cos 14x - \cos 8x),$$

$$-\sin 7x \sin x = \frac{1}{2}(\cos 8x - \cos 6x).$$

Cộng vế với vế ba đẳng thức trên, ta thấy biểu thức cần xét bằng 0 với mọi x.

50. a) Ta có

$\sin A = \cos B + \cos C \Leftrightarrow \sin A = 2\cos\frac{B+C}{2}\cos\frac{B-C}{2} \Leftrightarrow$

$\Leftrightarrow 2\sin\frac{A}{2}\left(\cos\frac{A}{2} - \cos\frac{B-C}{2}\right) = 0 \Leftrightarrow \cos\frac{A}{2} = \cos\frac{B-C}{2}$

($\sin\frac{A}{2} \neq 0$ vì $0 < A < \pi$).

Nhưng $0 < \frac{A}{2} < \frac{\pi}{2}$, $\left|\frac{B-C}{2}\right| < \frac{\pi}{2}$ nên $\cos\frac{A}{2} = \cos\frac{B-C}{2}$ khi và chỉ khi

$$\frac{A}{2} = \left|\frac{B-C}{2}\right|, \text{ tức là } A = |B-C|.$$

- Nếu $B > C$ thì $A = B - C$. Suy ra $B = \frac{\pi}{2}$.
- Nếu $B < C$ thì $A = C - B$. Suy ra $C = \frac{\pi}{2}$.

b) $\sin A = 2\sin B\cos C \Leftrightarrow \sin A = \sin(B+C) + \sin(B-C)$

$\Leftrightarrow \sin A = \sin(\pi - A) + \sin(B-C)$

$\Leftrightarrow \sin(B-C) = 0.$

Vì $0 \leq |B-C| < \pi$, nên $B - C = 0$. Vậy tam giác ABC cân tại A.

51. a) $\sin\alpha + \sin\beta + \sin\gamma = \sin\alpha + 2\sin\frac{\beta+\gamma}{2}\cos\frac{\beta-\gamma}{2}$

$= \sin\alpha + 2\sin\frac{\pi-\alpha}{2}\cos\frac{\beta-\gamma}{2} = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + 2\cos\frac{\alpha}{2}\cos\frac{\beta-\gamma}{2}$

$= 2\cos\frac{\alpha}{2}\left(\sin\frac{\alpha}{2} + \cos\frac{\beta-\gamma}{2}\right) = 2\cos\frac{\alpha}{2}\left[\sin\frac{\pi-(\beta+\gamma)}{2} + \cos\frac{\beta-\gamma}{2}\right]$

$= 2\cos\frac{\alpha}{2}\left(\cos\frac{\beta+\gamma}{2} + \cos\frac{\beta-\gamma}{2}\right) = 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}.$

b) $\cos\alpha + \cos\beta + \cos\gamma = \cos\alpha + 2\cos\frac{\beta+\gamma}{2}\cos\frac{\beta-\gamma}{2}$

$= \cos\alpha + 2\cos\frac{\pi-\alpha}{2}\cos\frac{\beta-\gamma}{2} = 1 - 2\sin^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\beta-\gamma}{2}$

$= 1 + 2\sin\frac{\alpha}{2}\left(-\sin\frac{\alpha}{2} + \cos\frac{\beta-\gamma}{2}\right)$

$= 1 + 2\sin\frac{\alpha}{2}\left[-\sin\frac{\pi-(\beta+\gamma)}{2} + \cos\frac{\beta-\gamma}{2}\right]$

$= 1 + 2\sin\frac{\alpha}{2}\left(-\cos\frac{\beta+\gamma}{2} + \cos\frac{\beta-\gamma}{2}\right) = 1 + 4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}.$

c) $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = \sin 2\alpha + 2\sin(\beta+\gamma)\cos(\beta-\gamma)$

$= 2\sin\alpha\,[\cos\alpha + \cos(\beta-\gamma)] = 2\sin\alpha\,[-\cos(\beta+\gamma) + \cos(\beta-\gamma)]$

$= 4\sin\alpha\sin\beta\sin\gamma.$

d) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \cos^2\alpha + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2}$

$= \cos^2\alpha + 1 + \frac{1}{2}(\cos 2\beta + \cos 2\gamma) = \cos^2\alpha + 1 + \cos(\beta+\gamma)\cos(\beta-\gamma)$

$= \cos^2\alpha + 1 - \cos\alpha\cos(\beta-\gamma) = 1 + \cos\alpha\,[\cos\alpha - \cos(\beta-\gamma)]$

$= 1 - \cos\alpha\,[\cos(\beta+\gamma) + \cos(\beta-\gamma)] = 1 - 2\cos\alpha\cos\beta\cos\gamma.$

52. a) $\tan\alpha + \tan\beta = \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta},$

$\tan\alpha - \tan\beta = \frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta} = \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}.$

b) $\frac{1}{\cos\alpha\cos 2\alpha} = \frac{\tan 2\alpha - \tan\alpha}{\sin\alpha},$

$\frac{1}{\cos 2\alpha\cos 3\alpha} = \frac{\tan 3\alpha - \tan 2\alpha}{\sin\alpha},$

...

$\frac{1}{\cos 7\alpha\cos 8\alpha} = \frac{\tan 8\alpha - \tan 7\alpha}{\sin\alpha}.$

Cộng vế với vế bảy đẳng thức trên, suy ra tổng đang xét bằng $\frac{\tan 8\alpha - \tan\alpha}{\sin\alpha}$.

53. $a = \cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$,

$b = \sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$,

nên suy ra $ab = 2\sin(\alpha+\beta)\cos^2\dfrac{\alpha-\beta}{2}$, $a^2 + b^2 = 4\cos^2\dfrac{\alpha-\beta}{2}$.

Từ đó $\sin(\alpha+\beta) = \dfrac{2ab}{a^2+b^2}$.

54. a) Gọi x là tầm xa của quỹ đạo thì $x > 0$ và $-\dfrac{gx^2}{2v^2\cos^2\alpha} + (\tan\alpha)x = 0$,

tức là $x = \dfrac{2v^2\sin\alpha\cos\alpha}{g} = \dfrac{v^2}{g}\sin 2\alpha$.

b) x đạt giá trị lớn nhất khi và chỉ khi $\sin 2\alpha = 1$, tức $\alpha = \dfrac{\pi}{4}$. Khi đó, $x = \dfrac{v^2}{g}$.

Với $v = 80$ m/s thì $x = \dfrac{v^2}{g} \approx \dfrac{80^2}{9{,}8} \approx 653$ (m).

(Đây là một bài tập có gắn với thực tiễn mà học sinh thường quan sát).