§3

2.12. a) $\frac{1}{4}$; b) $\frac{10^3}{10^{\log 5}} = \frac{10^3}{5} = 200$; c) $\frac{2}{3}$; d) 2.

2.13. a) $\log_7 \sqrt{36} - \log_7 14 - \log_7 21 = \log_7 \frac{1}{49} = -2$;

b) $\dfrac{\log_2 24 - \log_2 \sqrt{72}}{\log_3 18 - \log_3 \sqrt[3]{72}} = \dfrac{\log_2 2^{\frac{3}{2}}}{\log_3 3^{\frac{4}{3}}} = \dfrac{9}{8}$;

c) $\dfrac{\log_2 2^2 + \log_2\left(2^{\frac{1}{2}}.5^{\frac{1}{2}}\right)}{\log_2\left(2^2.5\right) + 3} = \dfrac{2 + \frac{1}{2} + \frac{1}{2}\log_2 5}{2 + 3 + \log_2 5} = \dfrac{1}{2}$.

2.14. a) $x = \dfrac{a^2}{b^3}$; b) $x = \dfrac{a^{\frac{2}{3}}}{b^{\frac{1}{5}}}$.

2.15. a) Ta có

• $a = \log_3 15 = \log_3(3.5) = \log_3 3 + \log_3 5 = 1 + \log_3 5$.

Suy ra $\log_3 5 = a - 1$.

• $b = \log_3 10 = \log_3(2.5) = \log_3 2 + \log_3 5$.

Suy ra $\log_3 2 = b - \log_3 5 = b - (a - 1) = b - a + 1$.

Do đó

$$\log_{\sqrt{3}} 50 = \log_{3^{\frac{1}{2}}}\left(2.5^2\right) = 2\log_3 2 + 4\log_3 5$$

$$= 2(b - a + 1) + 4(a - 1) = 2a + 2b - 2.$$

b) Ta có

$$\begin{aligned}\log_{140} 63 &= \log_{140}\left(3^2.7\right) = 2\log_{140} 3 + \log_{140} 7\\ &= \frac{2}{\log_3 140} + \frac{1}{\log_7 140} = \frac{2}{\log_3\left(2^2.5.7\right)} + \frac{1}{\log_7\left(2^2.5.7\right)}\\ &= \frac{2}{2\log_3 2 + \log_3 5 + \log_3 7} + \frac{1}{2\log_7 2 + \log_7 5 + 1}.\end{aligned}$$

Từ đề bài suy ra

$$\log_3 2 = \frac{1}{\log_2 3} = \frac{1}{a},$$

$$\log_7 5 = \log_7 2.\log_2 3.\log_3 5 = cab,$$

$$\log_3 7 = \frac{1}{\log_7 3} = \frac{1}{\log_7 2.\log_2 3} = \frac{1}{ca}.$$

Vậy $$\log_{140} 63 = \frac{2}{\frac{2}{a} + b + \frac{1}{ca}} + \frac{1}{2c + cab + 1} = \frac{2ac + 1}{abc + 2c + 1}.$$

2.16. a) $\log_3 \frac{6}{5} > \log_3 \frac{5}{6}$;

b) $\log_{\frac{1}{3}} 9 > \log_{\frac{1}{3}} 17$;

c) $\log_{\frac{1}{2}} \mathrm{e} > \log_{\frac{1}{2}} \pi$;

d) $\log_2 \frac{\sqrt{5}}{2} > \log_2 \frac{\sqrt{3}}{2}$.

2.17. a) Sử dụng tính chất $\log_a b.\log_b c = \log_a c$.

b) Sử dụng tính chất $\log_{a^k} b = \frac{1}{k}\log_a b$

và $1 + 2 + ... + n = \frac{n(n+1)}{2}$.