§3

4.17. *Đáp số :* a) $\frac{31}{13} - \frac{12}{13}i$; b) $\frac{27}{5} + \frac{9}{5}i$.

4.18. a) $x = \frac{(1+2i)(4+i)}{3+4i} = \frac{42}{25} + \frac{19}{25}i$; b) $x = \frac{-3+4i}{-5+2i} = \frac{23}{29} - \frac{14}{29}i$;

c) $x = \frac{-1+3i}{8-5i} = \frac{-23}{89} + \frac{19}{89}i$.

4.19. a) Giả sử $\frac{z_1}{z_2} = z$. Ta có $z_1 = z.z_2$, suy ra $\overline{z}_1 = \overline{z}.\overline{z}_2$ hay $\overline{z} = \frac{\overline{z}_1}{\overline{z}_2}$.

Vậy $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z}_1}{\overline{z}_2}$.

b) Tương tự, $|z_1| = |z.z_2| = |z|.|z_2|$ hay $|z| = \frac{|z_1|}{|z_2|}$. Vậy $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$.

4.20. a) Hiển nhiên $z \in \mathbb{R}$ thì $\overline{z} = z$. Ngược lại, giả sử $z = a + bi$ và $z = \overline{z}$. Từ đó suy ra $a + bi = a - bi$ và do đó $b = -b$ hay $b = 0$.

Vậy $z \in \mathbb{R}$.

b) Ta có $z = \frac{-3 - 2i\sqrt{3}}{\sqrt{2} + 3i} + \frac{-3 + 2i\sqrt{3}}{\sqrt{2} - 3i}$,

$$\text{suy ra } \bar{z} = \overline{\left(\frac{-3-2i\sqrt{3}}{\sqrt{2}+3i} + \frac{-3+2i\sqrt{3}}{\sqrt{2}-3i}\right)} = \overline{\left(\frac{-3-2i\sqrt{3}}{\sqrt{2}+3i}\right)} + \overline{\left(\frac{-3+2i\sqrt{3}}{\sqrt{2}-3i}\right)}$$

$$= \frac{\overline{-3-2i\sqrt{3}}}{\overline{\sqrt{2}+3i}} + \frac{\overline{-3+2i\sqrt{3}}}{\overline{\sqrt{2}-3i}} = \frac{-3+2i\sqrt{3}}{\sqrt{2}-3i} + \frac{-3-2i\sqrt{3}}{\sqrt{2}+3i} = z.$$

Vậy $z \in$ ¡ .

4.21. a) $\dfrac{1}{\sqrt{2}-i\sqrt{3}} = \dfrac{\sqrt{2}+i\sqrt{3}}{5} = \dfrac{\sqrt{2}}{5} + \dfrac{\sqrt{3}}{5}i$; b) $\dfrac{1}{i} = -i$;

c) $\dfrac{3-2i}{1+i\sqrt{5}} = \dfrac{(3-2i)(1-i\sqrt{5})}{6} = \dfrac{3-2\sqrt{5}}{6} - \dfrac{3\sqrt{5}+2}{6}i$

d) $\dfrac{1}{(3+i\sqrt{2})^2} = \dfrac{(3-i\sqrt{2})^2}{121} = \dfrac{7}{121} - \dfrac{6\sqrt{2}}{121}i.$