LỜI GIẢI – HƯỚNG DẪN – ĐÁP SỐ

1. *Đáp số :*

a) $-4 \approx -229^{\circ}10'59''$; b) $\dfrac{\pi}{13} \approx 13^{\circ}50'21''$; c) $\dfrac{4}{7} \approx 32^{\circ}44'26''$.

2. *Đáp số :*

a) $137^{\circ} \approx 2{,}391$; b) $-78^{\circ}35' \approx -1{,}371$; c) $26^{\circ} \approx 0{,}454$.

3. *Đáp số :*

a) $l \approx 33{,}66$ cm ; b) $l \approx 21{,}380$ cm ; c) $l \approx 33{,}333$ cm.

4. (h.63)

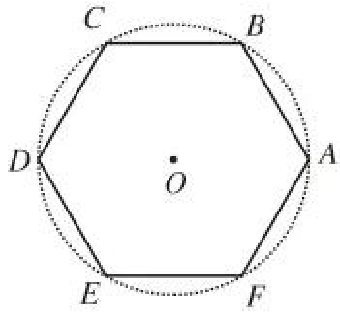
sđ $\overset{\curvearrowright}{AB} = \dfrac{\pi}{3} + k2\pi, k \in \mathbb{Z}$;

sđ $\overset{\curvearrowright}{AC} = \dfrac{2\pi}{3} + k2\pi, k \in \mathbb{Z}$;

sđ $\overset{\curvearrowright}{AD} = \pi + k2\pi, k \in \mathbb{Z}$;

sđ $\overset{\curvearrowright}{AE} = \dfrac{4\pi}{3} + k2\pi, k \in \mathbb{Z}$;

sđ $\overset{\curvearrowright}{AF} = \dfrac{5\pi}{3} + k2\pi, k \in \mathbb{Z}$.



Hình 63

5. Ta có sđ $\overset{\curvearrowright}{AB} = 15 + k2\pi, k \in \mathbb{Z}$.

$$15 + k2\pi < 0 \Leftrightarrow k < -\frac{15}{2\pi}.$$

Vậy với $k = -3$ ta được cung $\overset{\curvearrowright}{AB}$ có số đo âm lớn nhất là $15 - 6\pi$.

6. *Đáp số :*

a) $x = 0{,}4\pi$; $k = 6$; b) $x = \dfrac{\pi}{5}$; $k = -1$; c) $x = \dfrac{5\pi}{4}$, $k = 1$.

7. a) Với $\pi < \alpha < \dfrac{3\pi}{2}$ thì $\dfrac{\pi}{2} < \alpha - \dfrac{\pi}{2} < \pi$, do đó $\cos\left(\alpha - \dfrac{\pi}{2}\right) < 0$.

b) $\dfrac{3\pi}{2} < \dfrac{\pi}{2} + \alpha < 2\pi$ nên $\sin\left(\dfrac{\pi}{2} + \alpha\right) < 0$.

c) $0 < \dfrac{3\pi}{2} - \alpha < \dfrac{\pi}{2}$ nên $\tan\left(\dfrac{3\pi}{2} - \alpha\right) > 0$.

d) $2\pi < \alpha + \pi < \dfrac{5\pi}{2}$ nên $\cot(\alpha + \pi) > 0$.

8. a) $\sin\left(\alpha + \dfrac{\pi}{2}\right) = \sin\left(\dfrac{\pi}{2} - (-\alpha)\right) = \cos(-\alpha) = \cos\alpha$.

b) $\cos\left(\alpha + \dfrac{\pi}{2}\right) = \cos\left(\dfrac{\pi}{2} - (-\alpha)\right) = \sin(-\alpha) = -\sin\alpha$.

c) $\tan\left(\alpha + \frac{\pi}{2}\right) = \dfrac{\sin\left(\alpha + \frac{\pi}{2}\right)}{\cos\left(\alpha + \frac{\pi}{2}\right)} = \dfrac{\cos\alpha}{-\sin\alpha} = -\cot\alpha.$

d) $\cot\left(\alpha + \frac{\pi}{2}\right) = \dfrac{\cos\left(\alpha + \frac{\pi}{2}\right)}{\sin\left(\alpha + \frac{\pi}{2}\right)} = \dfrac{-\sin\alpha}{\cos\alpha} = -\tan\alpha.$

9. a) $\pi < \alpha < \frac{3\pi}{2} \Rightarrow \sin\alpha < 0.$

Vậy $\sin\alpha = -\sqrt{1-\cos^2\alpha} = -\sqrt{1-\frac{1}{16}} = -\frac{\sqrt{15}}{4},$

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \sqrt{15}, \cot\alpha = \dfrac{1}{\sqrt{15}}.$

b) $\frac{\pi}{2} < \alpha < \pi \Rightarrow \cos\alpha < 0.$

Vậy $\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-\frac{4}{9}} = \frac{-\sqrt{5}}{3},$

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = -\dfrac{2}{\sqrt{5}},\ \cot\alpha = -\dfrac{\sqrt{5}}{2}.$

c) $0 < \alpha < \frac{\pi}{2} \Rightarrow \cos\alpha > 0, \cos^2\alpha = \dfrac{1}{1+\tan^2\alpha}.$

Vậy $\cos\alpha = \dfrac{1}{\sqrt{1+\frac{49}{9}}} = \dfrac{3}{\sqrt{58}},$

$\sin\alpha = \cos\alpha\tan\alpha = \dfrac{7}{\sqrt{58}}, \cot\alpha = \dfrac{3}{7}.$

d) $\frac{3\pi}{2} < \alpha < 2\pi \Rightarrow \sin\alpha < 0, \sin^2\alpha = \dfrac{1}{1+\cot^2\alpha}.$

Vậy $\sin\alpha = -\dfrac{1}{\sqrt{1+\frac{196}{81}}} = -\dfrac{9}{\sqrt{277}},$

$\cos\alpha = \sin\alpha\cot\alpha = \dfrac{14}{\sqrt{277}},\quad \tan\alpha = \dfrac{1}{\cot\alpha} = -\dfrac{9}{14}.$

10. a) $\frac{\pi}{2} < \alpha < \pi \Rightarrow \cos\alpha < 0.$

Ta có $\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-\frac{9}{16}} = -\frac{\sqrt{7}}{4},$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = -\frac{3}{\sqrt{7}},\ \cot\alpha = -\frac{\sqrt{7}}{3};$$

Vậy $$A = \frac{-\frac{6}{\sqrt{7}} + \sqrt{7}}{-\frac{\sqrt{7}}{4} - \frac{3}{\sqrt{7}}} = -\frac{4}{19}.$$

b) $$B = \frac{\frac{7}{16} + \frac{7}{9}}{-\frac{3}{\sqrt{7}} + \frac{\sqrt{7}}{3}} = \frac{\frac{7 \times 25}{144}}{-\frac{2}{3\sqrt{7}}} = -\frac{175\sqrt{7}}{96}.$$

11. Vì $\pi < \alpha < \frac{3\pi}{2}$ nên $\cos\alpha < 0,\ \sin\alpha < 0$ và $\tan\alpha > 0.$

Ta có $\tan\alpha - 3\cot\alpha = 6 \Leftrightarrow \tan\alpha - \frac{3}{\tan\alpha} - 6 = 0$

$$\Leftrightarrow \tan^2\alpha - 6\tan\alpha - 3 = 0.$$

Vì $\tan\alpha > 0$ nên $\tan\alpha = 3 + 2\sqrt{3}.$

a) $\cos^2\alpha = \frac{1}{1+\tan^2\alpha} = \frac{1}{22+12\sqrt{3}}$

suy ra $\cos\alpha = -\frac{1}{\sqrt{22+12\sqrt{3}}},\ \sin\alpha = -\frac{3+2\sqrt{3}}{\sqrt{22+12\sqrt{3}}}.$

Vậy $\sin\alpha + \cos\alpha = -\frac{4+2\sqrt{3}}{\sqrt{22+12\sqrt{3}}}.$

b) $\dfrac{2\sin\alpha - \tan\alpha}{\cos\alpha + \cot\alpha} = \dfrac{\sin\alpha\left(2 - \dfrac{1}{\cos\alpha}\right)}{\cos\alpha\left(1 + \dfrac{1}{\sin\alpha}\right)}$

$= \tan\alpha.\dfrac{2\cos\alpha - 1}{\cos\alpha}.\dfrac{\sin\alpha}{\sin\alpha + 1} = \tan^2\alpha.\dfrac{2\cos\alpha - 1}{\sin\alpha + 1}$

$= (3 + 2\sqrt{3})^2.\dfrac{-\dfrac{2}{\sqrt{22 + 12\sqrt{3}}} - 1}{-\dfrac{3 + 2\sqrt{3}}{\sqrt{22 + 12\sqrt{3}}} + 1} = (21 + 12\sqrt{3}).\dfrac{2 + \sqrt{22 + 12\sqrt{3}}}{3 + 2\sqrt{3} - \sqrt{22 + 12\sqrt{3}}}.$

12. a) $\dfrac{\tan\alpha - \tan\beta}{\cot\beta - \cot\alpha} = \dfrac{\tan\alpha - \tan\beta}{\dfrac{1}{\tan\beta} - \dfrac{1}{\tan\alpha}} = \dfrac{\tan\alpha - \tan\beta}{\dfrac{\tan\alpha - \tan\beta}{\tan\alpha\tan\beta}} = \tan\alpha\tan\beta.$

b) $\tan 100^o + \dfrac{\sin 530^o}{1 + \sin 640^o} = \tan(90^o + 10^o) + \dfrac{\sin(360^o + 170^o)}{1 + \sin(720^o - 80^o)}$

$= -\cot 10^o + \dfrac{\sin 170^o}{1 - \sin 80^o} = -\dfrac{\cos 10^o}{\sin 10^o} + \dfrac{\sin 10^o}{1 - \cos 10^o}$

$= \dfrac{-\cos 10^o + \cos^2 10^o + \sin^2 10^o}{\sin 10^o(1 - \cos 10^o)} = \dfrac{1}{\sin 10^o}.$

c) $2(\sin^6 x + \cos^6 x) + 1 = 2(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x\cos^2 x + \cos^4 x) + 1$

$= 2(\sin^4 x + \cos^4 x) + 1 - 2\sin^2 x\cos^2 x$

$= 2(\sin^4 x + \cos^4 x) + (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$

$= 2(\sin^4 x + \cos^4 x) + (\sin^4 x + \cos^4 x)$

$= 3(\sin^4 x + \cos^4 x).$

13. a) $\tan^2\alpha + \cot^2\alpha = (\tan\alpha + \cot\alpha)^2 - 2\tan\alpha\cot\alpha = m^2 - 2$;

b) $\tan^3\alpha + \cot^3\alpha = (\tan\alpha + \cot\alpha)(\tan^2\alpha - \tan\alpha\cot\alpha + \cot^2\alpha)$

$= m(m^2 - 3)$.

14. a) $A = \tan(90^\circ - 72^\circ)\tan(360^\circ - 72^\circ) + \sin 32^\circ \sin(180^\circ - 32^\circ)$

$- \sin(360^\circ - 58^\circ)\sin(180^\circ - 58^\circ)$

$= \cot 72^\circ(-\tan 72^\circ) + \sin^2 32^\circ + \sin^2 58^\circ$

$= -1 + \sin^2 32^\circ + \cos^2 32^\circ$

$= -1 + 1 = 0.$

b) $$B = \frac{1 + (\sin^2\alpha + \cos^2\alpha)(\sin^2\alpha - \cos^2\alpha)}{1 - (\sin^2\alpha + \cos^2\alpha)(\sin^4\alpha - \sin^2\alpha\cos^2\alpha + \cos^4\alpha)}$$

$$= \frac{1 + \sin^2\alpha - \cos^2\alpha}{1 - [(\sin^2\alpha + \cos^2\alpha)^2 - 3\sin^2\alpha\cos^2\alpha)]}$$

$$= \frac{2\sin^2\alpha}{3\sin^2\alpha\cos^2\alpha} = \frac{2}{3}(1 + \tan^2\alpha).$$

15. Ta có

$$\frac{\sin\alpha + \tan\alpha}{\cos\alpha + \cot\alpha} = \frac{\sin\alpha\left(1 + \dfrac{1}{\cos\alpha}\right)}{\cos\alpha\left(1 + \dfrac{1}{\sin\alpha}\right)} = \frac{\sin^2\alpha(1 + \cos\alpha)}{\cos^2\alpha(1 + \sin\alpha)}.$$

Vì $1 + \cos\alpha \geq 0$ và $1 + \sin\alpha \geq 0$ cho nên biểu thức đã cho không thể có giá trị là một số âm.

16. Ta có

$$\sin\left(\alpha + \frac{\pi}{6}\right) - \cos\left(\alpha - \frac{2\pi}{3}\right)$$

$$= \sin\alpha\cos\frac{\pi}{6} + \cos\alpha\sin\frac{\pi}{6} - \cos\alpha\cos\frac{2\pi}{3} - \sin\alpha\sin\frac{2\pi}{3}$$

$$= \frac{\sqrt{3}}{2}\sin\alpha + \frac{1}{2}\cos\alpha + \frac{1}{2}\cos\alpha - \frac{\sqrt{3}}{2}\sin\alpha$$

$$= \cos\alpha = \frac{1}{3}.$$

17. Ta có

$$\cos\alpha = \sqrt{1-\frac{64}{289}} = \sqrt{\frac{225}{289}} = \frac{15}{17}\ ;\ \cos\beta = \sqrt{1-\frac{225}{289}} = \sqrt{\frac{64}{289}} = \frac{8}{17}.$$

Do đó

$$\begin{aligned}\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta\\ &= \frac{8}{17}\cdot\frac{8}{17} + \frac{15}{17}\cdot\frac{15}{17} = \frac{289}{289} = 1.\end{aligned}$$

Vì $0 < \alpha < \frac{\pi}{2}$ và $0 < \beta < \frac{\pi}{2}$ nên từ đó suy ra $\alpha + \beta = \frac{\pi}{2}\cdot$

18. a) $\sin 20^o + 2\sin 40^o - \sin 100^o = (\sin 20^o - \sin 100^o) + 2\sin 40^o$

$$= 2\cos 60^o \sin(-40^o) + 2\sin 40^o$$

$$= -\sin 40^o + 2\sin 40^o = \sin 40^o.$$

b) $$\frac{\sin(45^o+\alpha) - \cos(45^o+\alpha)}{\sin(45^o+\alpha) + \cos(45^o+\alpha)} = \frac{\sin(45^o+\alpha) - \sin(45^o-\alpha)}{\sin(45^o+\alpha) + \sin(45^o-\alpha)}$$

$$= \frac{2\cos 45^o \sin\alpha}{2\sin 45^o \cos\alpha} = \frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \tan\alpha.$$

c) $$A = \frac{3\cot^2 15^o - 1}{3 - \cot^2 15^o} = \frac{\cot^2 30^o \cot^2 15^o - 1}{\cot^2 30^o - \cot^2 15^o}$$

$$= \frac{\cot 30^o \cot 15^o + 1}{\cot 30^o - \cot 15^o}\cdot\frac{\cot 30^o \cot 15^o - 1}{\cot 30^o + \cot 15^o}.$$

Mặt khác ta có

$$\cot(\alpha+\beta) = \frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)} = \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\cos\beta + \cos\alpha\sin\beta}.$$

Chia cả tử và mẫu của biểu thức cho $\sin\alpha\sin\beta$ ta được

$$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}\cdot$$

Tương tự

$$\cot(\alpha - \beta) = \frac{\cot\alpha\cot\beta + 1}{\cot\beta - \cot\alpha}.$$

Do đó

$$A = \cot(15^o - 30^o)\cot(15^o + 30^o) = -\cot 15^o.$$

d) $\sin 200^o \sin 310^o + \cos 340^o \cos 50^o$

$$= \sin(180^o + 20^o)\sin(360^o - 50^o) + \cos(360^o - 20^o)\cos 50^o$$

$$= (-\sin 20^o)(-\sin 50^o) + \cos 20^o \cos 50^o$$

$$= \cos 50^o \cos 20^o + \sin 50^o \sin 20^o$$

$$= \cos(50^o - 20^o) = \frac{\sqrt{3}}{2}.$$

19. a) $\sin 6\alpha \cot 3\alpha - \cos 6\alpha = 2\sin 3\alpha \cos 3\alpha \cdot \frac{\cos 3\alpha}{\sin 3\alpha} - (2\cos^2 3\alpha - 1)$

$$= 2\cos^2 3\alpha - 2\cos^2 3\alpha + 1 = 1.$$

b) $[\tan(90^o - \alpha) - \cot(90^o + \alpha)]^2 - [\cot(180^o + \alpha) + \cot(270^o + \alpha)]^2$

$$= (\cot\alpha + \tan\alpha)^2 - (\cot\alpha - \tan\alpha)^2$$

$$= \cot^2\alpha + 2 + \tan^2\alpha - \cot^2\alpha + 2 - \tan^2\alpha = 4.$$

c) $(\tan\alpha - \tan\beta)\cot(\alpha - \beta) - \tan\alpha\tan\beta = \frac{\tan\alpha - \tan\beta}{\tan(\alpha - \beta)} - \tan\alpha\tan\beta$

$$= 1 + \tan\alpha\tan\beta - \tan\alpha\tan\beta = 1.$$

d) $\left(\cot\frac{\alpha}{3} - \tan\frac{\alpha}{3}\right)\tan\frac{2\alpha}{3} = \left(\frac{\cos\frac{\alpha}{3}}{\sin\frac{\alpha}{3}} - \frac{\sin\frac{\alpha}{3}}{\cos\frac{\alpha}{3}}\right)\frac{\sin\frac{2\alpha}{3}}{\cos\frac{2\alpha}{3}}$

$$= \frac{\cos^2\frac{\alpha}{3} - \sin^2\frac{\alpha}{3}}{\sin\frac{\alpha}{3}\cos\frac{\alpha}{3}} \cdot \frac{\sin\frac{2\alpha}{3}}{\cos\frac{2\alpha}{3}} = \frac{\cos\frac{2\alpha}{3}}{\frac{1}{2}\sin\frac{2\alpha}{3}} \cdot \frac{\sin\frac{2\alpha}{3}}{\cos\frac{2\alpha}{3}} = 2.$$

20. a) $\sin^4\frac{\pi}{16}+\sin^4\frac{3\pi}{16}+\sin^4\frac{5\pi}{16}+\sin^4\frac{7\pi}{16}$

$$=\left(\frac{1-\cos\frac{\pi}{8}}{2}\right)^2+\left(\frac{1-\cos\frac{3\pi}{8}}{2}\right)^2+\left(\frac{1-\cos\frac{5\pi}{8}}{2}\right)^2+\left(\frac{1-\cos\frac{7\pi}{8}}{2}\right)^2$$

$$=\frac{1}{4}\left(1-2\cos\frac{\pi}{8}+\cos^2\frac{\pi}{8}+1-2\cos\frac{3\pi}{8}+\cos^2\frac{3\pi}{8}+1-2\cos\frac{5\pi}{8}+\cos^2\frac{5\pi}{8}\right.$$

$$\left.+\,1-2\cos\frac{7\pi}{8}+\cos^2\frac{7\pi}{8}\right)$$

$$=1-\frac{1}{2}\left(\cos\frac{\pi}{8}+\cos\frac{3\pi}{8}+\cos\frac{5\pi}{8}+\cos\frac{7\pi}{8}\right)$$

$$+\frac{1}{4}\left(\frac{1+\cos\frac{\pi}{4}}{2}+\frac{1+\cos\frac{3\pi}{4}}{2}+\frac{1+\cos\frac{5\pi}{4}}{2}+\frac{1+\cos\frac{7\pi}{4}}{2}\right)$$

$$=1-\frac{1}{2}\left(\cos\frac{\pi}{8}+\cos\frac{3\pi}{8}-\cos\frac{3\pi}{8}-\cos\frac{\pi}{8}\right)+\frac{1}{8}\left(4+\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)$$

$$=\frac{3}{2}.$$

b) $\cot 7{,}5^\circ+\tan 67{,}5^\circ-\tan 7{,}5^\circ-\cot 67{,}5^\circ$

$$=\frac{\cos 7{,}5^\circ}{\sin 7{,}5^\circ}-\frac{\sin 7{,}5^\circ}{\cos 7{,}5^\circ}+\frac{\sin 67{,}5^\circ}{\cos 67{,}5^\circ}-\frac{\cos 67{,}5^\circ}{\sin 67{,}5^\circ}$$

$$=\frac{\cos^2 7{,}5^\circ-\sin^2 7{,}5^\circ}{\sin 7{,}5^\circ\cos 7{,}5^\circ}+\frac{\sin^2 67{,}5^\circ-\cos^2 67{,}5^\circ}{\sin 67{,}5^\circ\cos 67{,}5^\circ}$$

$$=\frac{\cos 15^\circ}{\frac{1}{2}\sin 15^\circ}-\frac{\cos 135^\circ}{\frac{1}{2}\sin 135^\circ}=\frac{2(\sin 135^\circ\cos 15^\circ-\cos 135^\circ\sin 15^\circ)}{\sin 15^\circ\sin 135^\circ}$$

$$= \frac{2\sin(135^\circ - 15^\circ)}{\sin(45^\circ - 30^\circ)\sin(180^\circ - 45^\circ)}$$

$$= \frac{2\sin 120^\circ}{(\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ)\sin 45^\circ}$$

$$= \frac{\sqrt{3}}{\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right).\frac{\sqrt{2}}{2}} = \frac{4\sqrt{3}}{\sqrt{3}-1} = 6 + 2\sqrt{3}.$$

21. a) $\dfrac{\sin 2\alpha + \sin\alpha}{1 + \cos 2\alpha + \cos\alpha} = \dfrac{\sin\alpha(2\cos\alpha + 1)}{2\cos^2\alpha + \cos\alpha}$

$$= \frac{\sin\alpha(2\cos\alpha + 1)}{\cos\alpha(2\cos\alpha + 1)} = \tan\alpha.$$

b) $\dfrac{4\sin^2\alpha}{1 - \cos^2\frac{\alpha}{2}} = \dfrac{16\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2}} = 16\cos^2\dfrac{\alpha}{2}.$

c) $\dfrac{1 + \cos\alpha - \sin\alpha}{1 - \cos\alpha - \sin\alpha} = \dfrac{2\cos^2\frac{\alpha}{2} - 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\sin^2\frac{\alpha}{2} - 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$

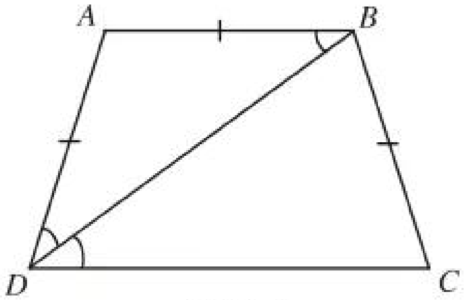
$$= \frac{2\cos\frac{\alpha}{2}\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)}{2\sin\frac{\alpha}{2}\left(\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}\right)} = -\cot\frac{\alpha}{2}.$$

d) $\dfrac{1 + \sin\alpha - 2\sin^2\left(45^\circ - \frac{\alpha}{2}\right)}{4\cos\frac{\alpha}{2}} = \dfrac{\sin\alpha + \cos(90^\circ - \alpha)}{4\cos\frac{\alpha}{2}}$

$$= \frac{\sin\alpha + \sin\alpha}{4\cos\frac{\alpha}{2}} = \frac{4\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\cos\frac{\alpha}{2}} = \sin\frac{\alpha}{2}.$$

22. Ta có (h.64)

$$\widehat{ABD} = \widehat{ADB}$$
$$\widehat{ABD} = \widehat{BDC}$$
$$\Rightarrow \quad \widehat{BDC} = \widehat{ADB}.$$



Hình 64

Suy ra $\widehat{BAD} = \pi - 2\widehat{BDC}$.

Từ đó ta có

$$\tan\widehat{BAD} = -\tan 2\widehat{BDC} = -\frac{2\tan\widehat{BDC}}{1-\tan^2\widehat{BDC}} = -\frac{2\cdot\frac{3}{4}}{1-\frac{9}{16}} = -\frac{3}{2}\cdot\frac{16}{7} = -\frac{24}{7}.$$

Vì $\frac{\pi}{2} < \widehat{BAD} < \pi$ nên $\cos\widehat{BAD} < 0$. Do đó

$$\cos\widehat{BAD} = -\frac{1}{\sqrt{1+\tan^2\widehat{BAD}}} = -\frac{1}{\sqrt{1+\frac{576}{49}}} = -\frac{7}{25}.$$

$$\sin\widehat{BAD} = \cos\widehat{BAD}\tan\widehat{BAD} = \frac{-7}{25}\cdot\frac{-24}{7} = \frac{24}{25}.$$

23. *Đáp số :*

a) Đúng ; b) Sai ; c) Sai ; d) Sai.

24. *Đáp số :*

a) Có ; b) Có ; c) Có ;

d) Không, vì $-1{,}2 < -1$. e) Không, vì $1{,}3 > 1$;

g) Không, vì $-2 < -1$.

25. a) $\sin 135^{\circ} > 0$, $\cos 135^{\circ} < 0$; b) $\sin 210^{\circ} < 0$, $\cos 210^{\circ} < 0$.

c) $\sin 334^{\circ} < 0$, $\cos 334^{\circ} > 0$;

d) $\sin 1280^{\circ} = \sin(3.360^{\circ} + 200^{\circ}) = \sin 200^{\circ} < 0$,

$\cos 1280^{\circ} = \cos 200^{\circ} < 0$;

e) $\sin(-235^{\circ}) = \sin(-180^{\circ} - 55^{\circ}) = -\sin(-55^{\circ}) = \sin 55^{\circ} > 0$,

$\cos(-235^{\circ}) < 0$;

g) $\sin(-1876^{\circ}) = \sin(-1800^{\circ} - 76^{\circ}) = \sin(-76^{\circ}) = -\sin 76^{\circ} < 0$,

$\cos(-1876^{\circ}) = \cos(-76^{\circ}) = \cos 76^{\circ} > 0$.

26. a) $\sin 220^{\circ} < \sin 10^{\circ} < \sin 40^{\circ} < \sin 90^{\circ}$.

b) $\cos 138^{\circ} < \cos 90^{\circ} < \cos 15^{\circ} < \cos 0^{\circ}$.

27. a) Ta có

$$\sin 110^{\circ} > 0 \,;\, \cos 130^{\circ} < 0 \,;\, \tan 30^{\circ} > 0 \,;\, \cot 320^{\circ} < 0,$$

do đó tích của chúng dương.

b) $\sin(-50^{\circ}) < 0$; $\tan 170^{\circ} < 0$; $\cos(-91^{\circ}) < 0$; $\sin 530^{\circ} > 0$,

do đó tích của chúng âm.

28. Vì các góc $\widehat{A}$, $\widehat{B}$, $\widehat{C}$ là góc trong tam giác ABC nên $\sin A > 0$, $\sin B > 0$, $\sin C > 0$. Do đó $\sin A + \sin B + \sin C > 0$.

29. a) $0 < \alpha < \frac{\pi}{2} \Rightarrow \cos\alpha > 0$, do đó

$$\cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - 0{,}36} = \sqrt{0{,}64} = 0{,}8.$$

$$\Rightarrow \quad \tan\alpha = \frac{3}{4},\ \cot\alpha = \frac{4}{3}.$$

b) $\frac{\pi}{2} < \alpha < \pi \Rightarrow \sin\alpha > 0$, do đó

$$\sin\alpha = \sqrt{1 - \cos^2\alpha} = \sqrt{1 - 0{,}49} = \sqrt{0{,}51} \approx 0{,}71.$$

Suy ra $\qquad \tan\alpha = -\frac{0{,}7}{0{,}71} \approx -0{,}98$; $\cot\alpha \approx -1{,}01$.

c) $\pi < \alpha < \frac{3\pi}{2} \Rightarrow \cos\alpha < 0$, do đó

$$\cos\alpha = -\frac{1}{\sqrt{1+\tan^2\alpha}} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}, \sin\alpha = -\frac{2\sqrt{5}}{5},$$

$$\cot\alpha = \frac{1}{2}.$$

d) $\frac{3\pi}{2} < \alpha < 2\pi \Rightarrow \sin\alpha < 0$, do đó

$$\sin\alpha = -\frac{1}{\sqrt{1+\cot^2\alpha}} = -\frac{1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}, \cos\alpha = \frac{3\sqrt{10}}{10},$$

$$\tan\alpha = -\frac{1}{3}.$$

30. a) $\sin(270^o - \alpha) = \sin(360^o - (90^o + \alpha)) = -\sin(90^o + \alpha) = -\cos\alpha.$

b) $\cos(270^o - \alpha) = \cos(360^o - (90^o + \alpha)) = \cos(90^o + \alpha) = -\sin\alpha.$

c) $\sin(270^o + \alpha) = \sin(360^o - (90^o - \alpha)) = -\sin(90^o - \alpha) = -\cos\alpha.$

d) $\cos(270^o + \alpha) = \cos(360^o - (90^o - \alpha)) = \cos(90^o - \alpha) = \sin\alpha.$

31. a) $\sin^2(180^o - \alpha) + \tan^2(180^o - \alpha)\tan^2(270^o + \alpha) + \sin(90^o + \alpha)\cos(\alpha - 360^o)$

$$= \sin^2\alpha + \tan^2\alpha\cot^2\alpha + \cos^2\alpha = 2.$$

b) $\frac{\cos(\alpha - 90^o)}{\sin(180^o - \alpha)} + \frac{\tan(\alpha - 180^o)\cos(180^o + \alpha)\sin(270^o + \alpha)}{\tan(270^o + \alpha)}$

$$= \frac{\sin\alpha}{\sin\alpha} + \frac{\tan\alpha(-\cos\alpha)(-\cos\alpha)}{-\cot\alpha} = 1 - \sin^2\alpha = \cos^2\alpha.$$

c) $\frac{\cos(-228^o)\cot 72^o}{\tan(-162^o)\sin 108^o} - \tan 18^o$

$$= \frac{\cos(72^o - 360^o)\cot 72^o}{\tan(18^o - 180^o)\sin(180^o - 72^o)} - \tan 18^o$$

$$= \frac{\cos 72^o \cot 72^o}{\tan 18^o \sin 72^o} - \tan 18^o$$

$$= \frac{\cot^2 72^o}{\tan 18^o} - \tan 18^o = \frac{\tan^2 18^o}{\tan 18^o} - \tan 18^o = 0.$$

d) Ta có $\sin 70^o = \cos 20^o$, $\sin 50^o = \cos 40^o$, $\sin 40^o = \cos 50^o$. Vì vậy

$$\frac{\sin 20^o \sin 30^o \sin 40^o \sin 50^o \sin 60^o \sin 70^o}{\cos 10^o \cos 50^o}$$

$$= \frac{\frac{1}{2}\cdot\frac{\sqrt{3}}{2}.\sin 20^o \cos 20^o \cos 50^o \cos 40^o}{\cos 10^o \cos 50^o} = \frac{\frac{1}{2}\cdot\frac{\sqrt{3}}{4}\sin 40^o \cos 40^o}{\cos 10^o}$$

$$= \frac{\frac{\sqrt{3}}{16}\sin 80^o}{\cos 10^o} = \frac{\sqrt{3}}{16}.$$

32. a) Với $0^o < \alpha < 90^o$ thì $0 < \cos\alpha < 1$ hay $\frac{1}{\cos\alpha} > 1$.

Nhân hai vế với $\sin\alpha > 0$ ta được $\tan\alpha > \sin\alpha$.

Vậy không có giá trị nào của α ($0^o < \alpha < 90^o$) để $\tan\alpha < \sin\alpha$.

b) Ta có $\sin\alpha + \cos\alpha > 0$ và $\sin\alpha\cos\alpha > 0$. Do đó

$$\begin{aligned}(\sin\alpha + \cos\alpha)^2 &= \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha \\ &= 1 + 2\sin\alpha\cos\alpha > 1.\end{aligned}$$

Từ đó suy ra $\sin\alpha + \cos\alpha > 1$.

33. a) Với $0 < \alpha < \frac{\pi}{2}$ thì $\cos\alpha > 0$, $\sin\alpha > 0$. Ta có

$$1 - \sin^2\alpha = \cos^2\alpha.$$

Mặt khác $\cos^2\alpha = (2\sin\alpha)^2 = 4\sin^2\alpha$ nên $5\sin^2\alpha = 1$ hay

$$\sin\alpha = \frac{1}{\sqrt{5}},\ \cos\alpha = \frac{2}{\sqrt{5}},\ \tan\alpha = \frac{1}{2},\ \cot\alpha = 2.$$

b) Với $\frac{\pi}{2} < \alpha < \pi$ thì $\sin\alpha > 0$, $\cos\alpha < 0$, $\tan\alpha < 0$.

Ta có

$$\cot\alpha = 4\tan\alpha \Rightarrow \frac{1}{\tan\alpha} = 4\tan\alpha$$

$$\Rightarrow \tan^2\alpha = \frac{1}{4} \Rightarrow \tan\alpha = -\frac{1}{2},\ \cot\alpha = -2$$

$$\cos\alpha = -\frac{1}{\sqrt{1+\tan^2\alpha}} = -\frac{1}{\sqrt{1+\frac{1}{4}}} = -\frac{2}{\sqrt{5}},\ \sin\alpha = \frac{1}{\sqrt{5}}.$$

34. a) $\tan 3\alpha - \tan 2\alpha - \tan\alpha = \tan(2\alpha + \alpha) - (\tan 2\alpha + \tan\alpha)$

$$= \frac{\tan 2\alpha + \tan\alpha}{1 - \tan 2\alpha \tan\alpha} - (\tan 2\alpha + \tan\alpha)$$

$$= (\tan 2\alpha + \tan\alpha)\left(\frac{1}{1 - \tan 2\alpha \tan\alpha} - 1\right)$$

$$= \frac{\tan 2\alpha + \tan\alpha}{1 - \tan 2\alpha \tan\alpha}(1 - 1 + \tan 2\alpha\ \tan\alpha) = \tan 3\alpha\ \tan 2\alpha\ \tan\alpha.$$

b) $\dfrac{4\tan\alpha(1 - \tan^2\alpha)}{(1 + \tan^2\alpha)^2} = \dfrac{2.2\tan\alpha}{1 + \tan^2\alpha}\cdot\dfrac{1 - \tan^2\alpha}{1 + \tan^2\alpha} = 2\sin 2\alpha\ \cos 2\alpha = \sin 4\alpha.$

c) $\dfrac{1 + \tan^4\alpha}{\tan^2\alpha + \cot^2\alpha} = \dfrac{1 + \tan^4\alpha}{\tan^2\alpha + \dfrac{1}{\tan^2\alpha}} = \dfrac{1 + \tan^4\alpha}{\dfrac{\tan^4\alpha + 1}{\tan^2\alpha}} = \tan^2\alpha.$

d) $\dfrac{\cos\alpha \sin(\alpha - 3) - \sin\alpha \cos(\alpha - 3)}{\cos\left(3 - \dfrac{\pi}{6}\right) - \dfrac{1}{2}\sin 3}$

$$= \frac{\sin(\alpha - 3 - \alpha)}{\cos 3 \cos\frac{\pi}{6} + \sin 3 \sin\frac{\pi}{6} - \frac{1}{2}\sin 3} = \frac{-\sin 3}{\frac{\sqrt{3}}{2}\cos 3} = -\frac{2\tan 3}{\sqrt{3}}.$$

35. a) $A = 2(\sin^2\alpha + \cos^2\alpha)(\sin^4\alpha + \cos^4\alpha - \sin^2\alpha\cos^2\alpha) - 3(\sin^4\alpha + \cos^4\alpha)$

$$= -\sin^4\alpha - \cos^4\alpha - 2\sin^2\alpha\cos^2\alpha$$

$$= -(\sin^2\alpha + \cos^2\alpha)^2 = -1.$$

b) $B = 4[(\sin^2\alpha + \cos^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha] - \cos 4\alpha$

$$= 4\left(1 - \frac{1}{2}\sin^2 2\alpha\right) - 1 + 2\sin^2 2\alpha = 3.$$

c) $C = 8(\cos^4\alpha - \sin^4\alpha)(\cos^4\alpha + \sin^4\alpha) - \cos 6\alpha - 7\cos 2\alpha$

$$= 8(\cos^2\alpha - \sin^2\alpha)(\cos^2\alpha + \sin^2\alpha)\,[(\cos^2\alpha + \sin^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha]$$

$$- \cos 6\alpha - 7\cos 2\alpha$$

$$= 8\cos 2\alpha\left(1 - \frac{1}{2}\sin^2 2\alpha\right) - \cos 6\alpha - 7\cos 2\alpha$$

$$= \cos 2\alpha - 4\cos 2\alpha \sin^2 2\alpha - \cos(4\alpha + 2\alpha)$$

$$= \cos 2\alpha - 2\sin 4\alpha \sin 2\alpha - \cos 4\alpha \cos 2\alpha + \sin 4\alpha \sin 2\alpha$$

$$= \cos 2\alpha - (\cos 4\alpha \cos 2\alpha + \sin 4\alpha \sin 2\alpha)$$

$$= \cos 2\alpha - \cos 2\alpha = 0.$$

36. a) $$\frac{\tan 2\alpha}{\tan 4\alpha - \tan 2\alpha} = \frac{\tan 2\alpha}{\dfrac{2\tan 2\alpha}{1 - \tan^2 2\alpha} - \tan 2\alpha} = \frac{1 - \tan^2 2\alpha}{1 + \tan^2 2\alpha} = \cos 4\alpha.$$

b) $$\sqrt{1 + \sin\alpha} - \sqrt{1 - \sin\alpha} = \sqrt{\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\right)^2} - \sqrt{\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)^2}$$

Vì $0 < \alpha < \frac{\pi}{2}$ nên $0 < \frac{\alpha}{2} < \frac{\pi}{4}$, suy ra $0 < \sin\frac{\alpha}{2} < \cos\frac{\alpha}{2}$.

Vậy

$$\sqrt{1 + \sin\alpha} - \sqrt{1 - \sin\alpha} = \cos\frac{\alpha}{2} + \sin\frac{\alpha}{2} - \left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)$$

$$= 2\sin\frac{\alpha}{2}.$$

c) $$\frac{3 - 4\cos 2\alpha + \cos 4\alpha}{3 + 4\cos 2\alpha + \cos 4\alpha} = \frac{3 - 4\cos 2\alpha + 2\cos^2 2\alpha - 1}{3 + 4\cos 2\alpha + 2\cos^2 2\alpha - 1}$$

$$= \frac{2(\cos^2 2\alpha - 2\cos 2\alpha + 1)}{2(\cos^2 2\alpha + 2\cos 2\alpha + 1)}$$

$$= \frac{(\cos 2\alpha - 1)^2}{(\cos 2\alpha + 1)^2} = \frac{(-2\sin^2\alpha)^2}{(2\cos^2\alpha)^2} = \tan^4\alpha.$$

d) $$\frac{\sin\alpha + \sin 3\alpha + \sin 5\alpha}{\cos\alpha + \cos 3\alpha + \cos 5\alpha} = \frac{(\sin 5\alpha + \sin\alpha) + \sin 3\alpha}{(\cos 5\alpha + \cos\alpha) + \cos 3\alpha}$$

$$= \frac{\sin 3\alpha(2\cos 2\alpha + 1)}{\cos 3\alpha(2\cos 2\alpha + 1)} = \tan 3\alpha.$$

37. ***Hướng dẫn.*** Giả thiết tam giác *ABC* không tù có nghĩa là các góc của tam giác nhỏ hơn hoặc $\frac{\pi}{2}$ và hiệu của hai góc cũng nằm trong khoảng từ $-\frac{\pi}{2}$ tới $\frac{\pi}{2}$. Do đó với $A \le \frac{\pi}{2}$ thì $\cos\frac{A}{2} \ge \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ còn với $-\frac{\pi}{2} < B - C < \frac{\pi}{2}$ thì $-\frac{\pi}{4} < \frac{B-C}{2} < \frac{\pi}{4}$ do đó $\cos\frac{B-C}{2} > 0$.

Giải

Ta có : $\cos 2A + 2\sqrt{2}(\cos B + \cos C) = 3$

$$\Leftrightarrow 1 - 2\sin^2 A + 4\sqrt{2}\cos\frac{B+C}{2}\cos\frac{B-C}{2} = 3$$

$$\Leftrightarrow 1 - 2\sin^2 A + 4\sqrt{2}\sin\frac{A}{2}\cos\frac{B-C}{2} = 3$$

$$\Leftrightarrow 2\sin^2 A - 4\sqrt{2}\sin\frac{A}{2}\cos\frac{B-C}{2} + 2 = 0$$

$$\Leftrightarrow \sin^2 A - 2\sqrt{2}\sin\frac{A}{2}\cos\frac{B-C}{2} + 1 = 0 \qquad (*)$$

Tam giác *ABC* không tù nên $\cos\frac{A}{2} \ge \frac{\sqrt{2}}{2}$, suy ra $\sqrt{2} \le 2\cos\frac{A}{2}$. Mặt khác, $\cos\frac{B-C}{2} > 0$ nên ta có :

$$2\sqrt{2}\sin\frac{A}{2}\cos\frac{B-C}{2} \le 4\sin\frac{A}{2}\cos\frac{A}{2}\cos\frac{B-C}{2}$$

hay

$$-2\sqrt{2}\sin\frac{A}{2}\cos\frac{B-C}{2} \ge -2\sin A\cos\frac{B-C}{2}.$$

Vì vậy vế trái của (*) $\ge \sin^2 A - 2\sin A\cos\frac{B-C}{2} + 1$

$$= \left(\sin A - \cos\frac{B-C}{2}\right)^2 - \cos^2\frac{B-C}{2} + 1$$

$$= \left(\sin A - \cos\frac{B-C}{2}\right)^2 + \sin^2\frac{B-C}{2} \ge 0.$$

Dấu đẳng thức xảy ra khi và chỉ khi $\begin{cases} B - C = 0 \\ \sin A = \cos\dfrac{B - C}{2} \end{cases} \Leftrightarrow \begin{cases} B = C \\ \sin A = 1 \end{cases}$

$\Leftrightarrow A = \dfrac{\pi}{2}, B = C = \dfrac{\pi}{4}.$

Vậy *ABC* là tam giác vuông cân.