Bài tập ôn chương V

1. a) $\cot^2 x - \dfrac{2x\cos x}{\sin^3 x}$;

b) $\dfrac{\cos\sqrt{x}\cos 3x + 6\sqrt{x}\sin\sqrt{x}\sin 3x}{2\sqrt{x}\cos^2 3x}$;

c) $6\cos 2x\,(\sin 2x + 8)^2$;

d) $6x^2\tan x + \dfrac{2x^3 - 5}{\cos^2 x}$.

2. a) $f(x)=\dfrac{1-\cos 3x}{3} \Rightarrow f'(x)=\sin 3x$. Ta có

$f'(x)=g(x) \Leftrightarrow (\cos 6x-1).\cot 3x=\sin 3x$ (điều kiện : $\sin 3x \neq 0 \Leftrightarrow \cos 3x \neq \pm 1$)

$\Leftrightarrow (\cos 6x-1).\cos 3x=\sin^2 3x$

$\Leftrightarrow (1-2\sin^2 3x-1).\cos 3x=\sin^2 3x$

$\Leftrightarrow \sin^2 3x.(2\cos 3x+1)=0$

$\Leftrightarrow \cos 3x=-\dfrac{1}{2}$ (vì $\sin 3x \neq 0$)

$\Leftrightarrow \cos 3x=\cos\dfrac{2\pi}{3} \Leftrightarrow 3x=\pm\dfrac{2\pi}{3}+k2\pi$

$\Leftrightarrow x=\pm\dfrac{2\pi}{9}+k\dfrac{2\pi}{3}$ $(k \in \mathbb{Z})$.

b) $f(x)=\dfrac{1}{2}\cos 2x \Rightarrow f'(x)=-\sin 2x$. Ta có

$f'(x)=g(x) \Leftrightarrow -\sin 2x=1-(\cos 3x+\sin 3x)^2$

$\Leftrightarrow 1+\sin 2x=(\cos 3x+\sin 3x)^2$

$\Leftrightarrow 1+\sin 2x=1+2\sin 3x\cos 3x$

$\Leftrightarrow \sin 6x-\sin 2x=0 \Leftrightarrow 2\cos 4x\sin 2x=0$

$$\Leftrightarrow \left[\begin{array}{l}\cos 4x=0\\ \sin 2x=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}4x=\dfrac{\pi}{2}+k\pi\\ 2x=n\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\dfrac{\pi}{8}+k\dfrac{\pi}{4}\\ x=n\dfrac{\pi}{2}\end{array}\right. \quad (k,n \in \mathbb{Z}).$$

c) $f(x)=\dfrac{1}{2}\sin 2x+5\cos x \Rightarrow f'(x)=\cos 2x-5\sin x$. Ta có

$f'(x)=g(x) \Leftrightarrow \cos 2x-5\sin x=3\sin^2 x+\dfrac{3}{1+\tan^2 x}$

$$\Leftrightarrow 5\sin x+\frac{3}{1+\tan^2 x}=\cos 2x-3\sin^2 x$$

$\Leftrightarrow 5\sin x + 3\cos^2 x = \cos^2 x - 4\sin^2 x \Leftrightarrow 5\sin x = -2\cos^2 x - 4\sin^2 x$

$\Leftrightarrow 5\sin x = -2 - 2\sin^2 x \Leftrightarrow 2\sin^2 x + 5\sin x + 2 = 0.$

Đặt $t = \sin x, t \in [-1;1]$, ta có phương trình $2t^2 + 5t + 2 = 0$.

Giải phương trình ta được $t = -\dfrac{1}{2}$ (loại $t = -2$).

Với $t = -\dfrac{1}{2}$, ta giải phương trình

$$\sin x = -\frac{1}{2} \Leftrightarrow \sin x = \sin\left(-\frac{\pi}{6}\right) \Leftrightarrow \left[\begin{array}{l} x = -\dfrac{\pi}{6} + k2\pi \\ x = \dfrac{7\pi}{6} + k2\pi \end{array}\right. \quad (k \in \mathbb{Z}).$$

3. a) $\dfrac{1}{8}$; b) 40 ; c) -2.

4. a) $f'(x) = 6(x^8 - x^5 + x^2 - x + 1)$

$$= 6x^2\left(x^6 - x^3 + \frac{1}{4}\right) + 3x^2 + 6\left(\frac{x^2}{4} - x + 1\right)$$

$$= 6x^2\left(x^3 - \frac{1}{2}\right)^2 + 3x^2 + 6\left(\frac{x}{2} - 1\right)^2 > 0, \forall x \in \mathbb{R}.$$

b) $f'(x) = 2 + \cos x > 0, \forall x \in \mathbb{R}$.

5. $f'(x) = 3x^2 + 2(a - 1)x + 2$.

$\Delta' = (a-1)^2 - 6 = a^2 - 2a - 5$. Ta phải có

$\Delta' < 0 \Leftrightarrow a^2 - 2a - 5 < 0 \Leftrightarrow 1 - \sqrt{6} < a < 1 + \sqrt{6}$.

Vậy $f'(x) > 0$ với mọi $x \in \mathbb{R}$ nếu $1 - \sqrt{6} < a < 1 + \sqrt{6}$.

6. $g'(x) = \cos x - 2a\cos 2x - \cos 3x + 2a$

$= 4a\sin^2 x + 2\sin x\sin 2x$

$= 4a\sin^2 x + 4\sin^2 x\cos x$

$= 4\sin^2 x\,(a + \cos x)$.

Rõ ràng với $a > 1$ thì $a + \cos x > 0$ và $\sin^2 x \geq 0$ với mọi $x \in \mathbb{R}$

nên với $a > 1$ thì $g'(x) \geq 0, \forall x \in \mathbb{R}$.

7. 2.

8. (1 ; 1).

9. 60°.

10. a) $c = 2, b = -1, d = 1$

$$\Rightarrow f(x) = x^3 - x^2 + 2x + 1 ;$$

b) $f'(x) = 3x^2 - 2x + 2 \Rightarrow f'(1) = 3$.

Phương trình tiếp tuyến tại $M(1 ; 3)$ là

$$y - 3 = 3(x - 1) \text{ hay } y = 3x.$$

c) $f'(\sin t) = 3\sin^2 t - 2\sin t + 2$.

$f'(\sin t) = 3 \Leftrightarrow 3\sin^2 t - 2\sin t - 1 = 0$

$$\Leftrightarrow \left[\begin{array}{l} \sin t = 1 \\ \sin t = -\dfrac{1}{3} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} t = \dfrac{\pi}{2} + k2\pi \\ t = \arcsin\left(-\dfrac{1}{3}\right) + k2\pi \\ t = \pi - \arcsin\left(-\dfrac{1}{3}\right) + k2\pi \quad (k \in \mathbb{Z}). \end{array}\right.$$

d) $f''(x) = 6x - 2 \Rightarrow f''(\cos t) = 6\cos t - 2$;

$g'(x) = 2x - 3 \Rightarrow g'(\sin t) = 2\sin t - 3$.

Vậy $\quad 6\cos t - 2 = 2\sin t - 3$

$\Leftrightarrow \quad 2\sin t - 6\cos t = 1$

$\Leftrightarrow \quad \sin t - 3\cos t = \dfrac{1}{2}$. Đặt $\tan\varphi = 3$, ta được

$\sin(t - \varphi) = \dfrac{1}{2}\cos\varphi = \alpha$. Suy ra

$$\left[\begin{array}{l} t = \varphi + \arcsin\alpha + k2\pi \\ t = \pi + \varphi - \arcsin\alpha + k2\pi \ (k \in \mathbb{Z}). \end{array}\right.$$

e) $$\lim_{z\to 0}\frac{f''(\sin 5z) + 2}{g'(\sin 3z) + 3} = \lim_{z\to 0}\frac{6\sin 5z}{2\sin 3z}$$

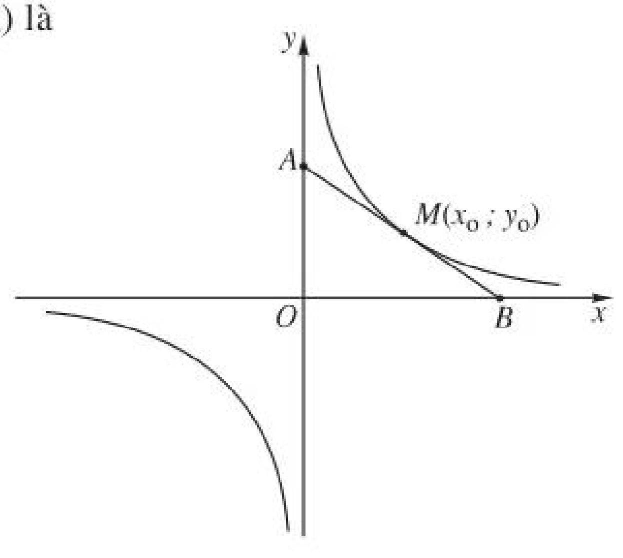
$$= 5\lim_{z\to 0}\frac{\dfrac{\sin 5z}{5z}}{\dfrac{\sin 3z}{3z}} = 5.$$

11. $y = \dfrac{a^2}{x} \Rightarrow y'(x_0) = -\dfrac{a^2}{x_0^2}$.

Phương trình tiếp tuyến tại $M(x_0 ; y_0)$ là

$$y - \frac{a^2}{x_0} = -\frac{a^2}{x_0^2}(x - x_0)$$

$$\Leftrightarrow \quad y = -\frac{a^2 x}{x_0^2} + \frac{2a^2}{x_0}.$$



Suy ra diện tích tam giác OAB là

$$S = \frac{1}{2}.\left|\frac{2a^2}{x_0}\right|.2|x_0| = 2a^2 = \text{const}.$$

12. HD : Chứng minh bằng quy nạp.